# PARALLEL APPLICATION ON HIGH PERFORMANCE COMPUTING PLATFORMS OF 3D BEM/FEM BASED COUPLING MODEL FOR DYNAMIC ANALYSIS OF SSI PROBLEMS

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**Abstract.** Implementation of an improved parallel computation algorithm into a coupled model based on Finite Element and Boundary Element Methods for analysis of three-dimensional Soil-Structure Interaction (SSI) problems on High-Performance Computing (HPC) platforms is presented. The model and the parallel computation algorithm are developed for the linear analysis of large-scale three-dimensional SSI problems. The finite element method is used for modeling the finite region and the structure, and the Boundary Element Method is used for modeling the soil extending to infinity. The parallelization of the model is performed by the calculation of the impedance coefficients on the interaction nodes between the near- and the far-fields. The performance of the parallel computation algorithm is represented by elapsed timing measurements according to the number of processors. The efficiency of the proposed parallel algorithm of the coupled model is validated with one numerical example that confirm the consistent accuracy and applicability of the parallel algorithm by considerable time saving for large-scale problems.

## 1 INTRODUCTION

In many fields of engineering, Finite Element Method (FEM) and the Boundary Element Method (BEM) are frequently applied analysis tools. Each one of these methods has its specific areas of applications. The FEM, is especially well suited for the analysis of problems involving in-homogeneities or non-linear behavior of solid bodies, [1, 2]. The BEM has advantages, if stress singularities on unbounded media are present. BEM has further advantages, if incident wave fields need to be considered in dynamic problems, [3-5]. Therefore, over the last 25 years, much progress has been made in finite/boundary element coupling methods to solve problems involving sub-regions with different characteristics. During this coupling, the respective advantages of both methods are used. In structural mechanics, these coupling methods are usually used for assessing the dynamic responses of stiff, heavy and embedded structures, such as nuclear reactors, gravity dams, tunnels, liquid-storage tanks and buildings with the soil media surrounding their foundations. Thus, soil-structure interaction (SSI), when incorporated, allows realistic prediction of the coupled behavior of the soil and structure. An arbitrarily shaped non-homogeneous body subjected to dynamic loads requires the use of the FEM and BEM together with unbounded boundary. BEM is a semi-analytical method and requires the fundamental solutions pertaining to the region, provided that the radiation condition at infinity is satisfied, [6-7].

Application of parallel computation to engineering problems is a relatively recent development that started approximately 15 years ago. The algorithms are widely used in engineering applications for large-scale computationally intensive problems. Widely used parallel algorithms are based on partitioning a computational domain and then assigning each partitioned domain to a separate computer processor, thus reaching a solution using many processors concurrently, [8, 9]. The parallel implementation of BEM codes have been studied by many prior researchers [10-14] and are well summarized in Davies[15]. More recent studies are found in Cunha and et al.[16], Bird and et al.[17], and Park and Heister[18]. Cunha and et al.[16] applied the standard and portable libraries for the parallelization of BEM codes; Bird and et al.[17] used a coupled BEM/scaled boundary FEM formulation to analyses linear elastic fracture mechanic problems; Park and Heister [18] proposed a parallelization procedure for the analysis of unsteady BEM problems. Most of these studies have worked on a structural problem or parallel implementation itself. However, in the study of Park and Heister[18] the simulation code for the free-surface problem has the unique dynamic grid characteristic.

The SBFEM is an alternative and effective method for modeling systems with finite and infinite extension having non-homogeneous material properties, [19-22]. Genes and Kocak[23, 24] applied the SBFEM to large-scale systems on high performance computing platforms. In this study, the linear equation solver of the system equations obtained from BEM for impedance analysis is used as portable library called The Scalable Linear Algebra Package (ScaLAPACK[1]) and the impedance analysis is also parallelized by distributing the impedance analysis work according to the number of the interaction nodes between the near- and far-field to the slave processors. Hence, the applicability of BEM/FEM coupling procedure to the high performance computing platforms for large-scale 3D SSI problems is demonstrated. The gained experience and the coded intelligent routines for the parallelization of SBFEM [23] are implemented to the parallelization of SBFEM/BEM/FEM [26] coupling and BEM/FEM coupling. In the proposed SSI model, where the boundary at the interface between the near-field (the structure and surrounding soil medium), and the medium which is extending to infinity, is modeled by the BEM. The structure and the surrounding soil medium are modeled by FEM. The dynamic stiffness matrix of the boundary is combined with the dynamic stiffness matrix of the finite medium by using the substructure method (SM). In this coupled model, best attributes of these two methods are combined.

Layered finite medium can be discretized by using a parametric soil model, where soil is composed of sub-layers, represented by some parameters as proposed by Kocak and Mengi[27]. However, in this study, to propose a more generic model, layered infinite medium is modeled by using 3D BEM formulation for linear elastodynamics[16]. The proposed model and parallel algorithm is verified by studying two examples that are analyzed by the coupled FEM/BEM and FE/SBFEM models. It is found that, depending on the number of processors

[1]The ScaLAPACK is a set of library for distributed memory MIMD (Multiple Instruction Multiple Data) parallel computers developed by the ScaLAPACK project, [25].

used, the proposed parallel algorithm can decrease the computation time considerably. The obtained results of the model are compared with the results given in the literature and good agreements are noted, [28-30]. Comparisons showed that the Parallelized Coupled FEM/BEM Model can be used in SSI analyses of large-scale structures efficiently and accurately. The results also demonstrate the importance and the advantages of using parallelization for SSI problems that are complex in geometry and have non-homogeneous unbounded media.

## 2 PHYSICAL MODELS AND NUMERICAL APPROACHES

### 2.1 Structural Dynamics with FEM

In this study, the dynamic response of structures is described by the equation of motion resulting from FEM formulation [1] in the time domain as,

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) \tag{2.1}$$

where; **M:** mass matrix, **C:** damping matrix denoting inner or structural damping, **K:** static stiffness matrix, **F**($t$): time dependent dynamic load acting on the structure caused by the external harmonic or transient vibrations or seismic excitation, $\ddot{\mathbf{U}}$ : acceleration vector, $\dot{\mathbf{U}}$ : velocity vector, and $\mathbf{U}$ : displacement vector. Equation of motion can be written as below in the frequency domain by ignoring the damping matrix and taking into account the structural damping,

$$\{(1+2iz)\mathbf{K} - \omega^2\mathbf{M}\}\ \mathbf{U}^f = \mathbf{F}^f \tag{2.2}$$

where; $z$: hysteretic damping, ω: frequency, $i$: imaginary number, and $f$: frequency space.

In this study, all the formulations are derived in frequency space. For the sake of simplicity, the superscript $f$ will be omitted. The term in curly brackets in Eq. (2.2) is referred to as the dynamic stiffness matrix and will be represented as **S**. For the finite region in Figure 1, Eq. (2.2) can be written in matrix form as,

$$\begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{si} \\ \mathbf{S}_{is} & \mathbf{S}_{ii} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s \\ \mathbf{F}_i \end{Bmatrix} \tag{2.3}$$

where; subscripts; $i$: interaction nodes, and $s$: non-interaction nodes.

### 2.2 Formulation of Elastodynamic Problems by Boundary Element Method

The detailed formulation of boundary element (BE) equation for elastodynamic problems in time and frequency domains is presented in the literature, [31-35]. The BE equation can be written as in Eq. (2.4) for the elastodynamic analysis of the 3D body shown in Figure 2 in Fourier transform domain.

$$\mathbf{c}\mathbf{u}(A) = \int_S \mathbf{G}(A,P)\mathbf{t}(P)dS + \int_S \mathbf{H}(A,P)\mathbf{u}(P)dS + \int_V \mathbf{G}(A,P)\mathbf{f}(P)dV \tag{2.4}$$

where; S indicates the surface; V volume of the body. **H** and **G** are the matrices obtained by the integration of the first and second fundamental solutions of BEM over the each boundary element. The **u**, **t** and **f** are the displacement, traction and volume force vectors, respectively.

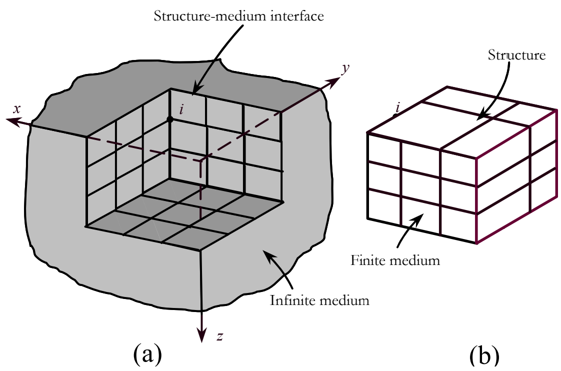


**Figure 1.** Fundamental concept of the coupled BEM/FEM based model **a)** Structure-medium interface modeled by BEM **b)** Finite medium and structure modeled by FEM

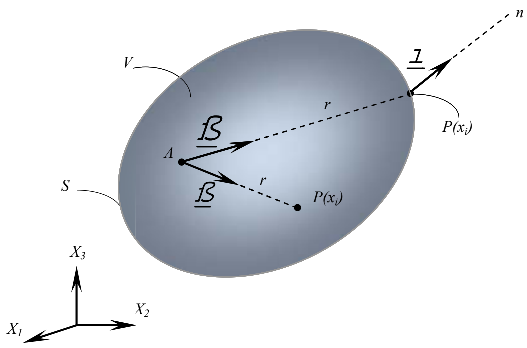

**Figure 2.** Three dimensional body for the BE formulation

Also, *A* and *P* indicates the used constant and integration points during the evaluation of the integrals. Equation (2.4) shows, the relation between the displacements of point *A* (Fig. 3), and the defined integrals on the body volume (*V*) and the boundary surface (*S*).

The BE equation given in Eq. (2.4) for the elastodynamic problems, can be used for the calculation of the unknown displacements and traction vectors on the boundary.

The resultant system of equation can be obtained by some manupilations of Eq. (2.4) as:

$$\mathbf{AX=F} \tag{2.5}$$

where; **A** is the coefficient matrix on the unknown quantities, and **F** is the force vector obtained from the multiplication of known quantities and their coefficient matrix.

The solution of Eq. (2.5) gives the unknowns on the boundary of the body. In this study, Eq. (2.5) is used for the calculation of the impedance coefficients on the interaction nodes between the structure and the far-field. The parallelization of the model is performed on the calculation of the impedance coefficients by using Eq. (2.5) and the parallel solution of Eq. (2.5) by using ScaLAPACK.

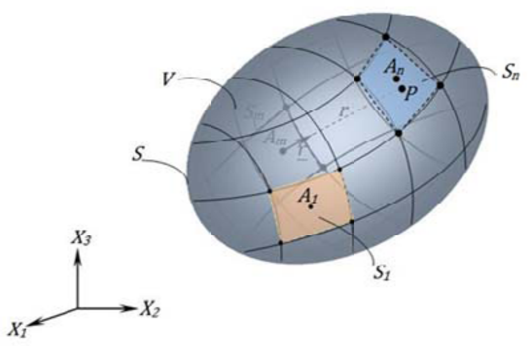

**Figure 3.** Discretized boundary of a body with boundary elements

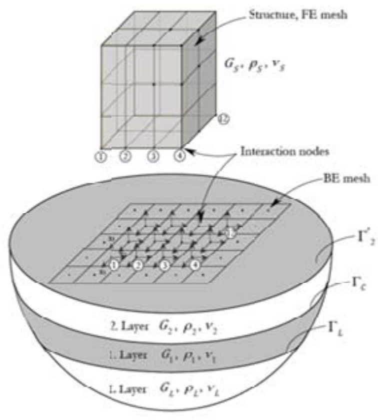


**Figure 4.** The boundary element mesh between the structure and the layered soil and the nodes used for impedance analysis

### 2.3 BE Formulation for Dynamic Analysis of Soil Medium with Two Layers

The BE formulation for the dynamic analysis of homogeneous soil medium, can be modified for the dynamic analysis of nonhomogeneous soil medium with two layers. The detailed formulation is given in the study of Tanrikulu [37]. The resultant system equation of the composite body composed of three materials can be obtained as:

$$\begin{bmatrix} \tilde{\mathbf{H}}_1 & \tilde{\mathbf{H}}_{1L} & \mathbf{0} & \tilde{\mathbf{H}}_{1c} & -\tilde{\mathbf{G}}_{1c} & -\tilde{\mathbf{G}}_{1L} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{H}}_2 & \tilde{\mathbf{H}}_{2c} & \tilde{\mathbf{G}}_{2c} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{H}}_L & \mathbf{0} & \mathbf{0} & \mathbf{0} & \tilde{\mathbf{G}}_L \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}_1 \\ \tilde{\mathbf{u}}_L \\ \tilde{\mathbf{u}}_2 \\ \tilde{\mathbf{u}}_c \\ \tilde{\mathbf{t}}_c \\ \tilde{\mathbf{t}}_L \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{G}}_1 & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{G}}_2 \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{t}}_1 \\ \tilde{\mathbf{t}}_2 \end{bmatrix} \tag{2.6}$$

The Eq. (2.6) can be written as a resultant system equation as in Eq. (2.5).

## 3 FEM/BEM COUPLING PROCEDURE FOR THE LAYERED SOIL AND PARALLELIZATION

The FEM/BEM coupling procedure for the layered soil (FE Eq. 2.3 and BE Eqs. 2.5 or 2.6), is not straightforward as the coupling procedure for the homogeneous soil [38, 26]. Therefore, with the help of the substructure method, impedance analysis must to be performed for the interaction nodes between the soil and the structure. The resulting impedance matrix is combined with the FEM Eq. (2.3). In this study, the impedance analysis is performed on the layered soil media by using the BEM. For the BEM analysis, an algorithm presented by Tanrikulu[37] for layered soil medium is used in the parallel algorithm proposed in this study. The definition of the impedance analysis can be explained on the model given in Figure 4. In other words, the impedance matrix is the inverse of the compliance matrix. The compliance matrix can be obtained by applying unit load in each degree of freedom of the interaction nodes to retrieve the displacements. The impedance matrix $\left(\mathbf{S}^{\infty}\right)$ is the contribution to the structure of the layered soil medium extending to infinity. The impedance matrix can be combined with the FEM equation by using the substructure method. The combined equation for the Coupled FEM/BEM model is given as:

$$\begin{bmatrix} \mathbf{S}_{ss} & \mathbf{S}_{si} \\ \mathbf{S}_{is} & \mathbf{S}_{ii} + \mathbf{S}^{\infty} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_s \\ \mathbf{U}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_s \\ \mathbf{F}_i \end{Bmatrix} \tag{3.1}$$

Impedance analysis for a SSI system with $n$ degrees of freedom of the interaction nodes between the structure and the layered soil, needs $n$ times repeated analysis of BEM equation. This is very time consuming analysis. Therefore, in this study a parallelization algorithm is proposed and applied. The proposed parallel algorithm calculates the impedance matrices for small frequency steps of an interval. If the number of nodes between the structure and the unbounded soil medium $m$, the total number of degrees of freedom on the interaction nodes is $n$=3$m$. This means that, at each frequency step, $n$ number of system of equations obtained by BEM on the interaction nodes have to be analyzed. The proposed parallel algorithm performs this analysis by partitioning the columns of the impedance matrix to the processors equally for the calculation of the impedance matrix (Fig. 5).

The parallel program assigns the load of each processor as,

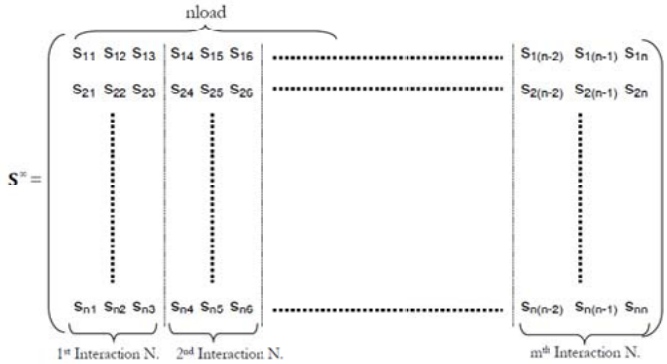


**Figure 5.** Partitioning the impedance matrix to the processor in parallel algorithm

$$nload = \frac{m}{nproc} \tag{3.2}$$

where, *m*: the number of interaction nodes, *nproc*: the number of slave processors. Consequently, each processor calculates the impedance vectors on *nload* number of nodes for each degree of freedom. The calculated impedance vectors are collected on the master processor and combined to obtain the full impedance matrix. The flowchart of the parallel program is presented in Figure 6. In the parallel program, one of the processors is assigned as the master and the others as the slaves. In the workflow of the parallel computation procedure, the master program first defines the computation load, then calculates the dynamic stiffness matrix of the FE region and defines the parameters of the far-field for BEM formulation. Then, it sends the data related to far-field and the computation load of each processor to the slave processors. Each slave processor computes the impedance coefficients at certain number of interaction nodes by using BEM. The BEM equation is solved by using ScaLAPACK library by the slave processor on sub-slave processors (Fig. 7). Figure 7 shows the communication scheme between the processors for a cluster has 16 processors. The impedance matrix ($\mathbf{S}^{\infty}$) is combined at the master processor by receiving the calculated columns of $\mathbf{S}^{\infty}$ from slave processors. The dynamic stiffness matrix of FE region is combined with impedance matrix by using the compatibility of the displacements and the equilibrium of the forces at interaction nodes as in Eq. (3.1) by the master processor. The solution of this linear system of equation is performed by LAPACK library [39].

Parallelization and fast matrix inversion are critical for dynamic analysis problems such as this as matrix inversion step tends to dominate computational time in most cases. The fully populated coefficient matrix provides a challenging problem for fast inversion schemes as one can not take advantage of any banded structures in general. Therefore, solving the linear system effectively may be a key factor to enhance the performance of the code. The ScaLAPACK is a set of library for distributed memory MIMD (Multiple Instruction Multiple Data) parallel computers developed by the ScaLAPACK project [25]. ScaLAPACK provides routines for dense and banded systems of linear equations; linear least squares problems, and eigenvalue and singular value problems. Since the ScaLAPACK routines are portable to any distributed memory computer using either Message Passing Interface (MPI) or a Parallel

Virtual Machine (PVM), the parallelization is directly carried out by applying the routine of ScaLAPACK on the linear system (**AX=F**) obtained from the BEM calculation.

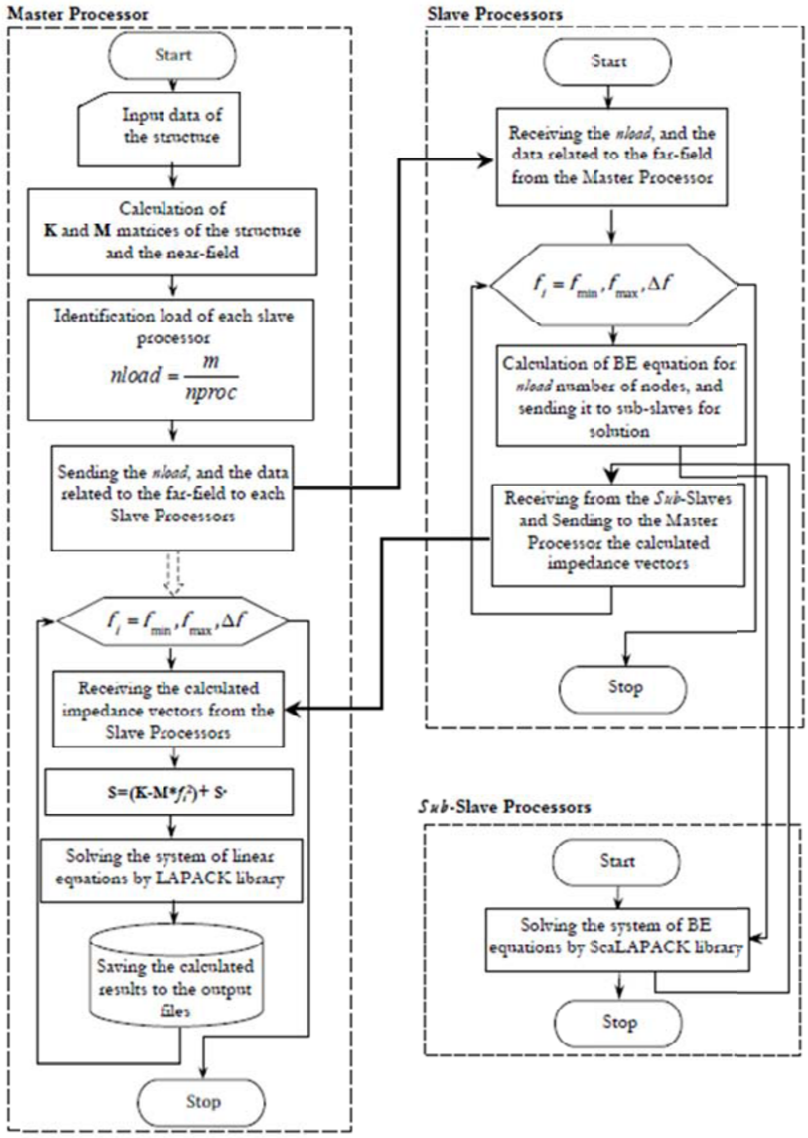


**Figure 6.** Parallel solution flowchart for the Coupled FE-BE Model of layered soil

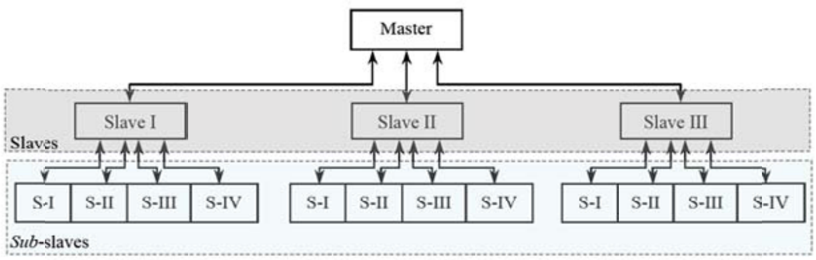


**Figure 7.** Communication scheme between the processors for a cluster with 16 processors

## 4 EFFICIENCY OF THE PARALLEL PROGRAM

In the parallel program, the master processor partition the BE computation load to the slave processors and collects the calculated impedance coefficients from the slaves. Next, the master processor adds the complete impedance matrix to the coefficient matrix of the FEM equation and solves it by using the LAPACK library. Initial testing of the efficiency of the improved program was performed by comparing the elapsed time for the BE meshes as shown in Figures 8a and 8b. The meshes have 384 and 1526 boundary elements. The durations are presented in Table 1 for the calculation of the first 5 frequencies of solution interval. As can be seen from these durations for different number of processors in Table 1, the proposed parallel algorithm decreases the computation time at least by a factor of approximately 15. By using the elapsed timing given in Table 1, the speedup curves are plotted in Figure 9. This figure exhibits the efficiency of the method with respect to the minimum number of processors, and as expected, when the number of processors increases to a certain number, the computation time decreases accordingly. It also can be seen from the Figure 9 that for larger problems the speedup is better, which can be explained by the problem becoming more computation bound, since for the platform used in this study, CPU speed is faster than that of message passing.

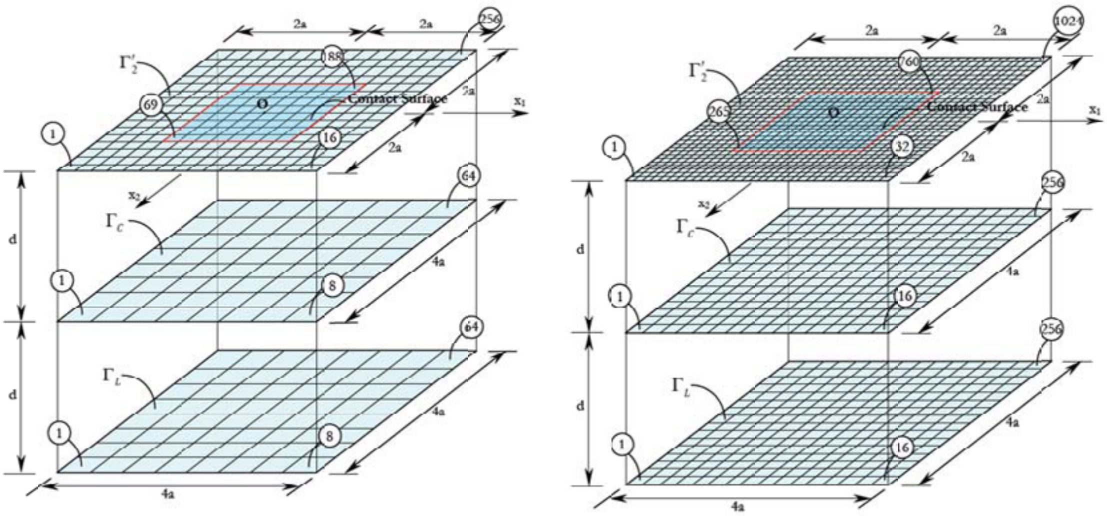


**Figure 8.** Boundary Element Meshes with 384 elements (Mesh I) and 1536 element (Mesh II)

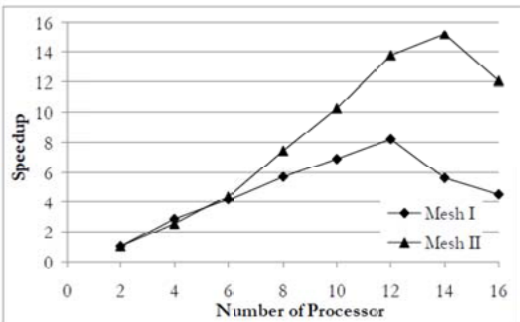


**Figure 9.** Speed-up curves for Mesh I and Mesh II

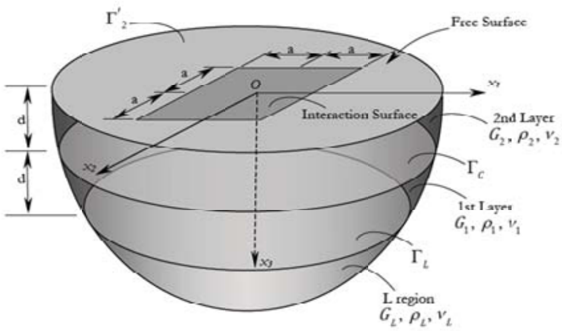


**Figure 10.** Rigid Square Foundation rest on layered soil media

**Table 1.** Timings for Mesh I and Mesh II according to number of processors

| | Timings according to number of processors (s) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **2** | **4** | **6** | **8** | **10** | **12** | **14** | **16** |
| **Mesh I** | 262 | 94 | 64 | 47 | 38 | 32 | 47 | 59 |
| **Mesh II** | 895 | 356 | 204 | 121 | 87 | 65 | 59 | 74 |

## 5 NUMERICAL EXAMPLES

In this section, one well known example problem is solved by the proposed parallel program for the presented SSI model and compared with other methods presented in the literature.

As stated before, in the present model, infinite nonhomogeneous soil region is modeled by boundary elements. The infinite regions, calculated by BEM, are plotted separately in order to demonstrate the interface between finite and infinite regions. During the analysis, the matching nodes of finite and infinite regions are automatically combined by the program.

### 5.1 Rigid Square Foundation on Elastic Nonhomogeneous Soil

The parallel solution algorithm and the presented model for SSI analysis in frequency domain are verified by comparing vertical, horizontal, and rocking impedances obtained for a rigid square foundation on layered nonhomogeneous soil (Fig. 10). The results are compared with those by Wong and Luco [28]. The rigid square foundation is considered completely connected to the soil media so that, the interaction surface between the structure and the soil medium is displaced compatibly with the rigid foundation. The origin of the coordinate system $X_1 X_2 X_3$ (O) is located at the center of the interaction surface (Fig. 10).

The BE mesh given in Figure 8a is used to compare the results obtained from Coupled FEM/BEM model in this study with those by Tanrikulu[37]. In this study, the impedance matrix is calculated by using the same BE solution algorithm presented by Tanrikulu[37]with the difference that, in this study, for the Coupled FEM/BEM model, the solution steps presented in the flowchart given in Figure 6 are used. In this problem the best speedup is obtained when 10 processors are used at the same time. For more consistent comparison with the literature, the BE mesh given by Tanrikulu[37] is also used. The horizontal, vertical and rocking impedances for the layered medium with different Poisson's ratios and material properties are presented in Figures 11-14. As seen in Figures 11 and 12, the present model demonstrates quite good match with the others, Wong and Luco[28]. In addition, Figure 13 shows the applicability of the model to layered soils.

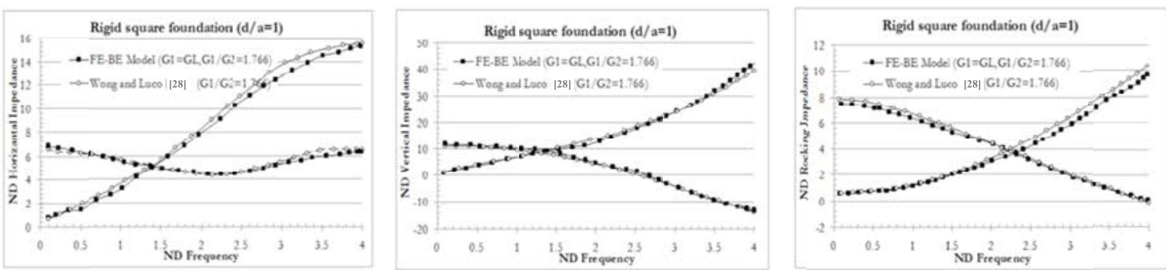


**Figure 11.** H, V and R impedance-frequency for three elastic materials ($\nu_1 = \nu_2 = \nu_L = 0.45$)

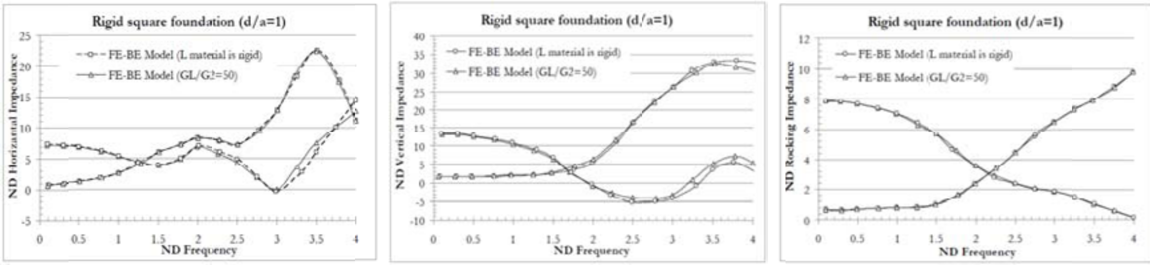


**Figure 12.** H, V and R impedance-frequency for two layers on rigid bedrock ($\nu_1 = \nu_2 = \nu_L = 0.45$)

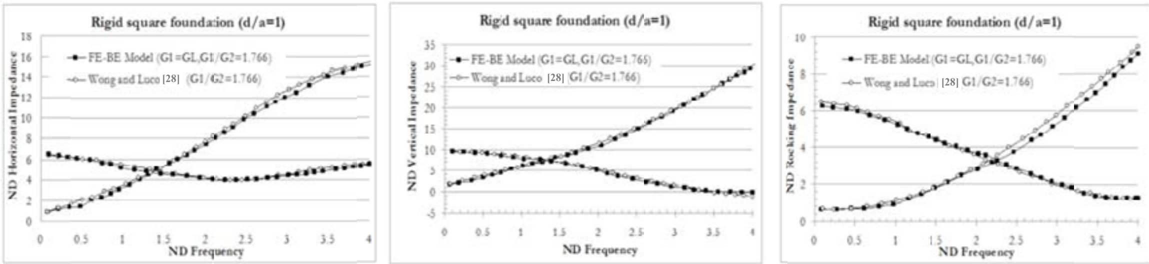


**Figure 13.** H, V and R impedance-frequency for three elastic materials ($\nu_1 = \nu_2 = \nu_L = 0.33$)

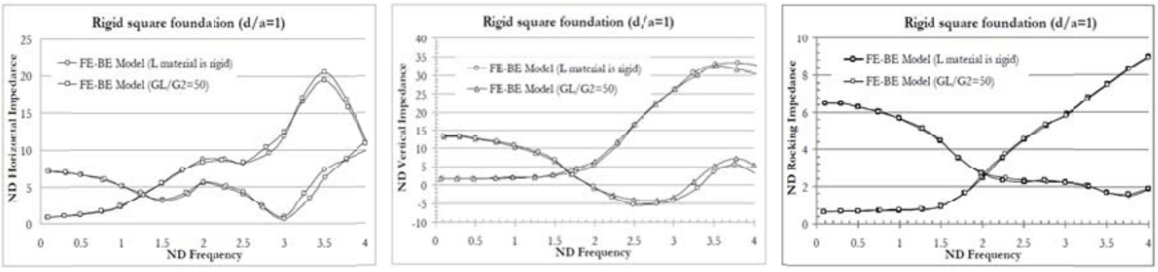


**Figure 14.** H, V and R impedance-frequency for two layers on rigid bedrock ($\nu_1 = \nu_2 = \nu_L = 0.33$)

# 6 SUMMARY AND CONCLUSIONS

In this article, a computational model is presented for large-scale SSI analysis of layered media using the Parallelized Coupled FEM/BEM Model. In the proposed model, the finite region, which might be considered as the structure, is modeled by the FEM. On the soil-structure interface, the boundary at the bottom of the finite media which is extending to infinity is modeled by the BEM. The analyses are conducted in the frequency domain.

Dynamic stiffness matrices pertaining to related regions of the SSI system are calculated by FEM and BEM, and combined by using sub-structuring method. Depending on the scale of the problem, the number of the slave processors can be determined by the user for the impedance analysis of the boundary by using BEM. The impedance matrix obtained by BEM for the contribution of the far-field is symmetric and the summation with the FEM equation is easy. Two example problems are solved to verify and investigate the applicability of the model and the coded parallel algorithm. It is found that, depending on the number of processers used, the proposed parallel algorithm can decrease the computation time at least by a factor of approximately 15 for the given example meshes. The obtained results of the model are compared with the results given in the literature and good agreements are noted. Comparisons showed that the Parallelized Coupled FEM/BEM Model can be used in SSI analyses of large-scale structures efficiently and accurately. The results also demonstrate the importance and the advantages of using parallelized coupled models for analyzing complex structures and non-homogeneous unbounded media. The proposed model and the developed program can be implemented to the non-linear analysis of SSI problems under transient or seismic loads.

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## References

[1] Bathe, K.J. *Finite element procedures.* Englewood Cliffs, NJ, Prentice-Hall; (1996).
[2] Zienkiewicz, O.C. and Taylor, R.L. *The finite element method.* Fifth ed. Oxford, Butterworth-Heinemann; (2000).
[3] Kane, J.H. *Boundary element analysis in engineering continuum mechanics.* Englewood Cliffs, NJ, Prentice-Hall; (1994).
[4] Bonnet, M. *Boundary integral equation methods for solids and fluids*. Chichester, Wiley; (1995).
[5] Dominquez, J. *Boundary elements in dynamics, Southampton*. Computational Mechanics Publications; (1993).
[6] Von Estorff, O., Pais, A.L. and Kausel, E. Some observations on time domain and frequency domain boundary elements. *International Journal for Numerical Methods in Engineering* (1990) **29**:785-800.
[7] Von Estorff, O. and Firuziaan, M. Coupled BEM/FEM approach for non-linear soil-structure interaction. *Engineering Analysis with Boundary Elements* (2000) **24**:715-725.
[8] Kocak, S. and Akay, H.U. Parallel schur complement method for large-scale systems on distributed memory computers. *Applied Mathematical Modelling* (2001) **25**:873-886.
[9] Farhat, C., Wilson, E. and Powell, G. Solution of finite element systems on concurrent processing computers. *Engineering with Computers* (1987) **2**:157-165.
[10] Symm, G.T. Boundary elements on a distributed array processor. *Engineering Analysis with Boundary Elements* (1984) **1(3)**:162–165.
[11] Davies, A.J. The boundary element method on the icl dap. *Parallel Computing* (1988) **8(1/3)**:335–43.
[12] Davies, A.J. Fine-grained parallel boundary elements. *Engineering Analysis with Boundary Elements* (1997) **19(1)**:13–6.
[13] Lobry, J. and Manneback, P. Parallel mr-bem using scalapack. *Engineering Analysis with Boundary Elements* (1997) **19(1)**:41–48.
[14] Song, S.W. and Baddour, R.E. Parallel processing for boundary element computations on distributed systems. *Engineering Analysis with Boundary Elements* (1997) **19(1)**:73–84.
[15] Davies, A.J. Parallel implementations of the boundary element method. *Computers & Mathematics with Applications* (1996) **31(6)**:33–40.
[16] Cunha, M.T.F., Telles, J.C.F., Coutinho, A.L.G.A. and Panetta, J. On the parallelization of boundary element codes using standard and portable libraries. *Engineering Analysis with Boundary Elements* (2004) **28(7)**:893–902.
[17] Bird, G.E., Trevelyan, J. and Augarde, C.E. A coupled BEM/scaled boundary FEM formulation for accurate computations in linear elastic fracture mechanics. *Engineering Analysis with Boundary Elements* (2010) **34**:599-610.

[18] Park, K.S. and Heister, S.D. On the parallelization of unsteady BEM problems with variable mesh size. *Engineering Analysis with Boundary Elements* (2010);34:289-296.
[19] Wolf, J.P. and Song, C. *Finite-element modelling of unbounded media.* Wiley and Sons Press, England; (1996).
[20] Wolf, J.P. *The scaled boundary finite element method.* Wiley and Sons Press, England; (2003).
[21] Deeks, A.J. and Wolf, J.P. Semi-analytical solution of Laplace's equation in non-equilibrating unbounded problems. *Computers and Structures* (2003) 81:1525-1537.
[22] Ekevid, T. and Wiberg, N.E. Wave propagation related to high-speed train: A scaled boundary FE-approach for unbounded domains. *Computer Methods in Applied Mechanics and Engineering* (2002) **191**:3947-3964.
[23] Genes, M.C. and Kocak, S. A combined finite element based soil-structure interaction model for large-scale systems and applications on parallel platforms. *Engineering Structures* (2002) **24(9)**:1119-1131.
[24] Genes, M.C. and Kocak, S. Parallel Treatment of Bulirsch-Stoer Integration Scheme for Soil-Structure Interaction Problems. In: Pala S et al., editors. *Fifth international congress on advances in civil engineering.* Istanbul: Istanbul Technical University; (2002).
[25] Blackford, L.S., Choi, J., Cleary, A., D'Azevedo, E., Demmel, J., Dhillon, I., et al. ScaLAPACK users' guide. *Society for Industrial and Applied Mathematics*; (1997).
[26] Genes, M.C. Dynamic analysis of large-scale SSI systems for layered unbounded media via a parallelized coupled finite-element/boundary-element/scaled boundary finite-element model, *Engineering Analysis with Boundary Elements* (2012) **36**:845-857.
[27] Kocak S, Mengi Y. A simple soil structure interaction model. *Applied Mathematical Modelling* (2000) **24**:607–635.
[28] Wong, H.L., Luco, J.E. Tables of impedance functions for square foundations on layered medium. *Soil Dynamics and Earthquake Engineering* (1985) **4(2)**:64-81.
[29] Huang, C.F.D. *Dynamic analysis of 3D massive foundation by time domain BEM*. Ph.D. Thesis. SC: University of South Caroline, Columbia; (1993).
[30] Rizos, D.C. and Wang, Z. Coupled BEM-FEM solutions for direct time domain soil-structure interaction analysis. *Engineering Analysis with Boundary Element* (2002) **26**:877-888.
[31] Brebbia, C.A. Dominguez J. *Boundary elements an introductory course*, Computational Mechanics Publications, Southampton; (1989).
[32] Banerjee, P.K. *The boundary element methods in engineering.* McGraw-Hill Book Company, London; (1994).
[33] Manolis, G.D. and Beskos, D.E. *Boundary element method in elastodynamics.* Unwin Hyman, London;(1987).
[34] Mengi, Y., Tanrikulu, A.H. and Tanrikulu A.K. *Boundary element method for elastic media an introduction.* Middle East Technical University Press, Ankara; (1994).
[35] Trevelyan, J. *Boundary elements for engineers, theory and applications.* Computational Mechanics Publications, Southampton; (1994).
[36] Press, W.H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T. *Numerical Recipes.* Cambridge University Press, Cambridge; (1988).
[37] Tanrikulu, A.H. *A boundary element model with nonlocal boundary conditions for dynamic analysis of a two-phase composite.* Ph.D. Thesis. Cukurova University, Engineering and Applied Science Institute, Adana; (1999).
[38] Genes, M.C. and Kocak, S. Dynamic soil–structure interaction analysis of layered unbounded media via a coupled finite element/boundary element/scaled boundary finite element model. *International Journal for Numerical Methods in Engineering* (2005) **62**:798–823.
[39] Anderson, E., Bai, Z., Bischof, C., Blackford, S., Demmel, J., Dongarra, J., Croz, J.D., Greenbaum, A., Hammarling, S., McKenney, A. and Sorensen, D. *LAPACK users guide*, 3rd ed. PA: SIAM, Philadelphia; (1999).